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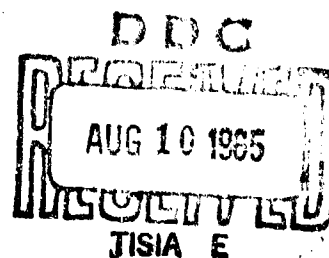
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# DESIGN CONSIDERATIONS AND ANALYSIS OF A COMPLEX MODULUS APPARATUS

R. L. ADKINS

TECHNICAL REPORT AFML-TR-65-104

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AIR FORCE MATERIALS LABORATORY  
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AFML-TR-65-104

**DESIGN CONSIDERATIONS AND ANALYSIS  
OF A COMPLEX MODULUS APPARATUS**

*R. L. ADKINS*

FOREWORD

This report was prepared by the Strength and Dynamics Branch, Metals and Ceramics Division, under Project No. 7351, "Metallic Materials", Task No. 735106, "Behavior of Metals". This research work was conducted in the Air Force Materials Laboratory, Research and Technology Division, Wright-Patterson Air Force Base, Ohio by R. L. Adkins.

This work began in December 1962 and was completed in August 1964. Manuscript released by author January 1965 for publication as an RTD Technical Report.

This report was formerly presented as a thesis in partial fulfillment of the requirements for the Degree of Master of Science at The Ohio State University. The cooperation of the Strength and Dynamics Branch in allowing the results of this study to be applied to the thesis is gratefully acknowledged. Further acknowledgement is made to Dr. D. L. G. Jones, W. J. Trapp, J. P. Henderson, and E. K. Tashima for their assistance and helpful discussions.

This technical report has been reviewed and is approved.



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ABSTRACT

An analytical assessment is made of an apparatus designed for the measurement of the complex shear modulus of viscoelastic materials. It is shown that simple measurements of amplitude ratio and phase angle are sufficient to determine the complex modulus at any point over a wide range of frequency and dynamic strain. Data obtained for butyl rubber and some experimental elastomers are presented.

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## SYMBOLS

A	specimen area
b	specimen thickness
c	velocity of shear waves in an elastic material = $\sqrt{G/\rho}$
f	the frequency of vibration
G	elastic shear modulus
$G^*$	complex shear modulus = $G' + i G'' = G_0 \exp(i\theta)$
$G'$	storage modulus in shear = $G_0 \cos \theta$
$G''$	loss modulus in shear = $G_0 \sin \theta$
$G_0$	magnitude of the complex shear modulus
$\theta$	loss angle in shear = $\arctan (G'' / G')$
i	$\sqrt{-1}$
Q	displacement amplitude of lower block
$\bar{Q}$	displacement amplitude of upper block or free mass
R	ratio, $\bar{Q}/Q$
u	displacement in the x direction
W	weight of the free mass the center of gravity of which is at the top of the specimen
x,y	rectangular coordinate axes
$\omega$	circular frequency = $2\pi f$
$\phi$	phase angle between the lower and upper blocks
$\tau$	shear stress
$\rho$	mass density of the specimen material
$\dot{h}$	$\frac{dh}{dt}$ (h)
H	$(\omega^2 M b)/A$
g	acceleration of gravity = $386 \text{ in./sec}^2$
k	$\frac{\omega}{c} = \frac{\omega}{\sqrt{G/\rho}}$
$k^*$	$\frac{\omega}{\sqrt{G^*/\rho}}$

## INTRODUCTION

The solution of problems in continuum mechanics wherein it is desired to represent a material which has time dependent mechanical behavior may be obtained by the use of the theory of linear viscoelasticity. Although the usefulness of this theory is restricted to situations involving small deformations, it is virtually indispensable as a first approximation when dealing with materials exhibiting stress or strain rate dependent behavior. This is completely analogous to the first approximation afforded by the classical theory of elasticity for so-called linear or Hookean solids.

Of the several theoretically equivalent means of describing the stress-strain behavior of linear viscoelastic solids, the "complex modulus" approach is the simplest to use from the point of view of an experimental determination of the properties of such materials. A rational introduction of the complex modulus concept is given by Snowdon (Reference 1). Data obtained from complex modulus experiments, under sinusoidal loading conditions, can be carried over to any of the other descriptions of the stress-strain behavior and used in the solution of various problems. Details of the connecting mathematical transformations may be found in Gross (Reference 2).

Although the complete characterization of mechanical properties of an isotropic, viscoelastic solid requires the determination of the two complex moduli in shear and dilatation, this work is limited to devising an experiment for measuring the complex modulus in shear alone.

The procedure adopted for this determination was to measure the response of the test material to a sinusoidal stimulus and compare the results with a simple analysis of the system. Obviously a candidate test for the determination of complex shear modulus must include a specimen for which the geometry, boundary conditions, and stress field are not only experimentally realizable but are also amenable to a relatively accurate analysis within the framework of the theory of linear viscoelasticity. It is also desirable that the dynamic response of the test specimen be dependent only on the complex shear modulus since, generally, it is possible to make a sufficient number of measurements to define only one complex modulus in any one type of test.

The test described herein satisfies to a reasonable degree, the previously stated conditions. Specifically, this work concerns the response of a sheet of test material bonded to the upper surface of a rigid metal plate and having a rigid mass bonded to its upper surface. The plate under the specimen is then forced in a steady sinusoidal motion.

## ANALYSIS OF THE RESPONSE OF THE TEST SPECIMEN

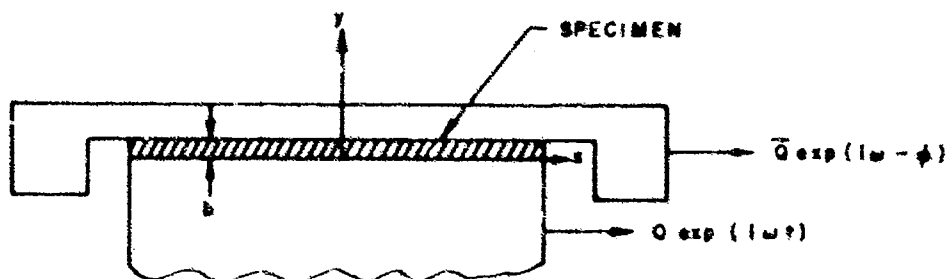


Figure 1. Schematic of Test Specimen

The specimen under test is shown schematically in Figure 1. The  $x, y$  coordinate axes are fixed in space. The displacement amplitude of the forced lower plate and the free mass are  $Q$  and  $\bar{Q}$  respectively and the phase angle by which the forced plate leads the free mass is  $\phi$ .

The effects of the stress free edges of the material in shear were neglected in this analysis. A state of pure shear was assumed. The upper and lower bounds of the errors introduced as a result of this assumption can be computed from the work of Read (Reference 3). According to Read, the correction to the stiffness is a function of Poisson's ratio and the length-to-thickness ratio of the specimen. If the material is assumed to be incompressible, then for a length-to-thickness ratio of 24, the decrease in stiffness due to allowance for the edge effects is between 0.67 percent and 1.12 percent. This apparatus was designed with constant specimen length of three inches and the thickness of the specimens could usually be made less than one eighth of an inch.

It was also assumed that the blocks to which the specimen is bonded are rigid and the static compressive forces due to gravity are neglected.

In the initial analysis the specimen material will be considered to be elastic. For pure shear all displacements vanish except  $u$ , the displacement in the  $x$  direction. The equations of motion reduce to the wave equation

$$G \left( \frac{\partial^2 u}{\partial y^2} \right) = \rho \left( \frac{\partial^2 u}{\partial t^2} \right)$$

or

$$\frac{\partial^2 u}{\partial y^2} = \frac{1}{c^2} \frac{\partial^2 u}{\partial t^2}$$

where  $c = \sqrt{\frac{G}{\rho}}$  is the velocity of shear waves in the specimen material.

For the steady state solution  $u(y,t) = F(y) \exp(i\omega t)$ , the equation of motion becomes

$$\begin{aligned} \frac{d^2 F}{dy^2} \exp(i\omega t) &= -\frac{\omega^2}{c^2} F \exp(i\omega t) \\ \frac{d^2 F}{dy^2} + \frac{\omega^2}{c^2} F &= 0 \end{aligned}$$

The general solution of this equation is

$$F(y) = B \sin(ky) + C \cos(ky) \quad (1)$$

where  $B$  and  $C$  are unknown constants to be determined from the boundary conditions and  $k = \omega/c$ . Then

$$u(y,t) = \left[ B \sin(ky) + C \cos(ky) \right] \exp(i\omega t) \quad (2)$$

The boundary conditions are:

1.  $u(0,t) = Q \exp(i\omega t)$ ;

this gives  $C = Q$  and  $F(y) = B \sin(ky) + Q \cos(ky)$

2. The shear force on the top of the specimen must oppose the inertia of the free mass or

$$A \tau_{y=b} = -M \ddot{u}_{y=b}$$

$$AG(\partial u / \partial y)_{y=b} = -\omega^2 M F(b) \exp(i\omega t)$$

$$AGk = [B \cos(kb) - Q \sin(kb)] = \omega^2 M [B \sin(kb) + Q \cos(kb)]$$

$$B = \frac{AGk \sin(kb) + \omega^2 M \cos(kb)}{AGk \cos(kb) - \omega^2 M \sin(kb)} Q$$

Substituting these results into Equation 1 yields

$$F(y) = Q \left[ \frac{AGk \cos(kb - ky) - \omega^2 M \sin(kb - ky)}{AGk \cos(kb) - \omega^2 M \sin(kb)} \right] \quad (3)$$

$$u(y,t) = Q \left[ \frac{AGk \cos(kb - ky) - \omega^2 M \sin(kb - ky)}{AGk \cos(kb) - \omega^2 M \sin(kb)} \right] \exp(i\omega t) \quad (4)$$

$$u(b,t) = Q \left[ \frac{AGk}{AGk \cos(kb) - \omega^2 M \sin(kb)} \right] \exp(i\omega t) \\ = Q \frac{1}{\cos(kb) - (\omega^2 M) / (AGk) \sin(kb)} \exp(i\omega t) \quad (5)$$

Now  $u(b,t)$  is measured as  $\bar{Q} \exp(i\omega t - \phi)$  as in Figure 1. Inserting this into Equation 5 yields

$$\frac{\bar{Q}}{Q} \exp(-i\phi) = \frac{1}{\cos(kb) - (\omega^2 M) / (AGk) \sin(kb)} \quad (6)$$

Thus far it has been assumed that  $G$  is real, that is, that the specimen material is purely elastic. The formal solution of viscoelastic problems has been greatly simplified by the use of the correspondence principle between elastic and viscoelastic problems. This principle was first noted by Alfrey (Reference 4) for incompressible materials and Lee (Reference 5) for compressible materials.

The Alfrey-Lee correspondence principle states that the Laplace transformed viscoelastic solution is obtained directly from the solution to an associated elastic problem by merely replacing the elastic constants by certain functions of the Laplace transform parameter depending on the nature of the stress-strain relations. The character of the associated elastic problem is established by Laplace transforming the differential equation and boundary and initial conditions defining the viscoelastic problem. Further proof of the correspondence principle is given by Read (Reference 6) and Berry (Reference 7). Berry also states that for a steady state sinusoidal stimulus the solution to a viscoelastic problem is obtained from that of the corresponding elastic problem by the substitution of complex moduli which are functions of frequency for the elastic moduli or constants.

As a consequence of the previous discussion, the response of the free mass on top of the viscoelastic medium is obtained directly from Equation 5 by replacing the elastic shear modulus with the complex shear modulus  $G^*$ .

$$G^* = G' + iG'' = G_0 \exp(i\theta) \quad \text{where } \theta = \arctan \left( \frac{G''}{G'} \right).$$

Equation 6 becomes

$$\frac{\bar{Q}}{Q} \exp(-i\phi) = \frac{1}{\cos(k^*b) - (\omega^2 M)/(AG^*k^*) \sin(k^*b)} \quad (7)$$

where

$$k^* = \frac{\omega}{\sqrt{G^*/\rho}} = \frac{\omega}{\sqrt{G_0/\rho}} \exp(-i\theta/2) = k_0 \exp(-i\theta/2)$$

If  $(k^*b)$  is small in magnitude and the quantities  $\cos(k^*b)$  and  $\sin(k^*b)$  are approximated by the first term of their respective Maclaurin series, the following approximation is obtained for Equation 7:

$$\frac{\bar{Q}}{Q} \exp(-i\phi) = R \exp(-i\phi) = \frac{1}{1 - (\omega^2 Mb)/(G^*A)} \quad (8)$$

The same result would be obtained if the specimen inertia were neglected in the original equation of motion. It is desirable to be able to use this approximate solution for the response because it can easily be solved explicitly for the complex modulus  $G^*$ . It is difficult to compute the error resulting from this approximation for the general case. In the Appendix a computation of the error for specific values of the material constants, specimen geometry, and excitation frequency are obtained. On the basis of this approximation, the solution of Equation 8 for  $G^*$  is:

$$G^* = \frac{\omega^2 Mb/A}{1 - 1/R \exp(i\phi)} = \frac{\omega^2 Mb/A}{(1 - 1/R \cos\phi) - i 1/R \sin\phi}$$

Therefore:

$$G_o = \frac{\omega^2 Mb/A}{\left[ (1/R)^2 - 2/R \cos \phi + 1 \right]^{1/2}}$$

and

$$\begin{aligned} \tan \theta &= \frac{1/R \sin \phi}{1 - 1/R \cos \phi} \\ &= \frac{\sin \phi}{R - \cos \phi} \end{aligned}$$

For steady state sinusoidal motion, the amplitude ratio of, and the phase angle between, the lower and upper blocks or plates will be the same for displacement, velocity, and acceleration signals. Which of these three quantities are measured depends primarily on the frequency at which the test is performed.  $R$  will now be redefined as any one of these three kinematic ratios.  $H$  will also be defined as equal to  $\omega^2 Mb/A$ .

The final formulas for  $G_o$  and  $\tan \theta$  are:

$$G_o = \frac{H}{(1/R^2 - 2/R \cos \phi + 1)^{1/2}} \quad (9)$$

and

$$\tan \theta = \frac{\sin \phi}{R - \cos \phi} \quad (10)$$

## DESIGN CONSIDERATIONS AND DEVELOPMENT

In the analysis an approximation was made which depended on the magnitude of  $k^*b$  being small.

$$|k^*b| = k_o b = \frac{\omega b}{\sqrt{G_o/\rho}} \ll 1$$

therefore

$$b \ll \frac{\sqrt{G_o/\rho}}{\omega}$$

Typical values of these material constants and frequency are:

$$G_o = 900 \text{ psi}, \rho = 10^{-4} \text{ mass units**/in.}^3, \text{ and } f = 500 \text{ cps or}$$

$$\omega = 1000\pi \text{ radians/sec. For these values}$$

$$\frac{\sqrt{G_o/\rho}}{\omega} = \frac{3}{\pi} \approx 1$$

Therefore  $b \leq 0.1$  in. should satisfy the inequality.

The mass of the upper block must be calculated. To permit accurate calculations, it is seen from Equations 9 and 10 that  $R$  must be different from unity and  $\phi$  different from zero. These differences are greatest near the resonance of the specimen-free mass system, where the resonance is defined here as the maximum of the amplitude ratio,  $R$ .

From Equation 8 it can easily be shown that  $R$  is a maximum for the frequency

$$\omega = \left[ (G'A/b)/M \right]^{1/2}.$$

This particular value will be called the natural circular frequency frequency of the specimen-mass system. Note that this natural frequency does not change with damping since it is only a function of the storage modulus. At this natural frequency,

$$M = \frac{G'A}{b\omega_n^2}. \quad (11)$$

The area of the specimen will be fixed at six square inches (three inches long in the direction of vibration and two inches wide). Choosing  $\cos \theta$  as unity and using the typical values of the other constants in relation to Equation 11, we have

$$M = 0.00547 \text{ mass units**}$$

$$W = Mg = 2.1 \text{ lb.}$$

Another consideration in designing this mass was to locate the center of gravity at the top edge of the specimen so that the stress distribution was simple shear. The final design for the mass is shown in Figure 2.

\*\*\*\*\*

\*\*No name has been assigned to the unit of mass in the inch-pound-second system of units. In this paper this unit will simply be called a mass unit.

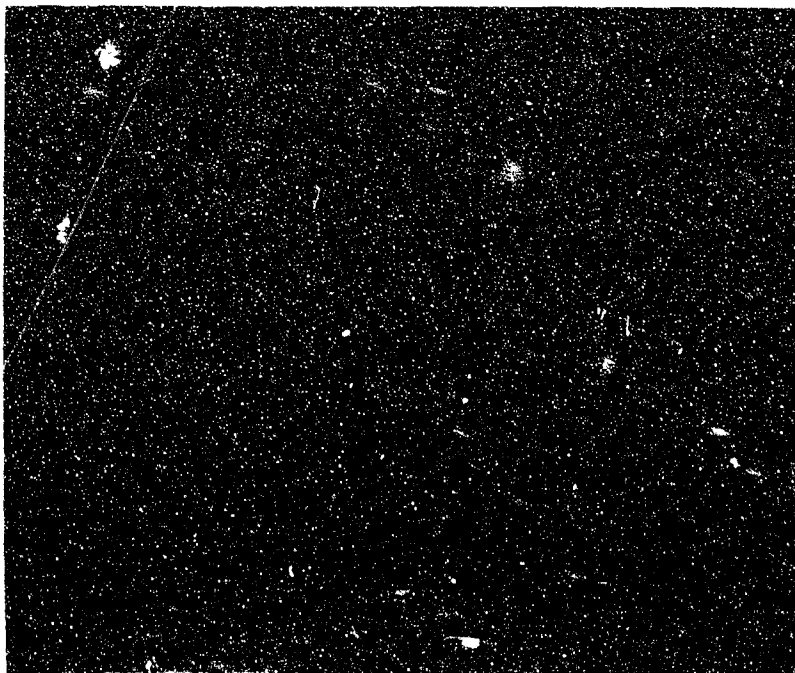


Figure 2. Free Mass Including Accelerometer and Dummy Mass

#### SPECIMEN PREPARATION AND TEST PROCEDURE

The metal plates above and below the specimen are of brass. Brass was primarily chosen because many elastomers will bond directly to brass without a secondary adhesive. When possible, the specimen was bonded to the plates while the specimen material underwent a specified curing cycle. These plates were attached to the other parts of the apparatus with machine screws. A mold, not shown, was designed to be compatible with the plates and has been used in specimen preparation. A photograph of a "ready for test" specimen is shown in Figure 3.

Before starting any test, it was necessary to find the specimen thickness and the weight of the free mass, including that of the accelerometer and dummy. The lower plate was mounted on a lightweight slip table which was rigidly connected to the armature of a 100 force pound electromagnetic exciter. The entire apparatus was mounted on a seismic concrete block as shown in Figure 4.

While the tests were being conducted, the accelerations of the upper and lower plates were measured by accelerometers and read out on vibration meters. Phase differences between the two accelerometers were measured by a precision phase meter. The two accelerometer signals were continuously observed on a dual trace oscilloscope to ensure that they were not highly distorted when data was taken. Frequency was measured by an electronic digital counter.

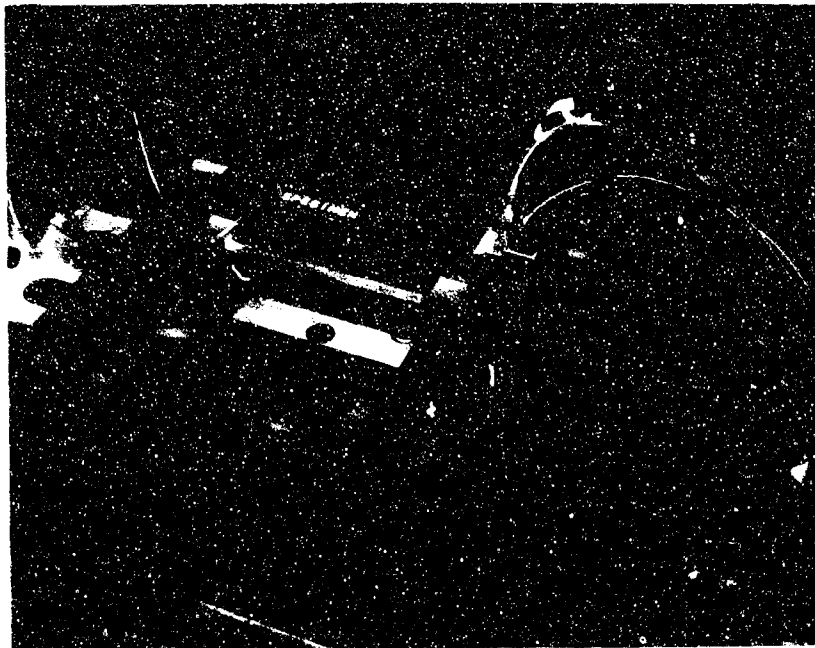


Figure 3. A "Ready for Test" Specimen



Figure 4. View of Complex Modulus Apparatus Including the Slip Table, Electromagnetic Exciter, and Top of the Seismic Concrete Block

## DISCUSSION OF RESULTS

The results plotted in Figures 5 and 6 show the typical frequency and temperature dependence of the properties of viscoelastic materials. Although the measurements of Figure 6 were taken at ambient room temperature, it was necessary to maintain this temperature to within  $0.1^{\circ}\text{C}$  to ensure adequate reproducibility of the measurements.

Figure 7 shows data for a silicone rubber material, the properties of which were essentially independent of temperature variations in the vicinity of room temperature. The properties were, however, found to be dependent on the amplitude of shear strain. This indicates that the real material is nonlinear. The calculated material properties for this real material, at each value of strain amplitude, may be called the viscoelastic constants of an "equivalent" linear viscoelastic material.

Data on a sample of butyl rubber of the same compounding formulation as that of Figure 5, and taken on a Maxwell machine (Reference 8) at frequencies up to 100 cps, was available for comparison. This machine measures the complex modulus in tension-compression rather than shear. To reduce these tension-compression moduli to shear-moduli, it was assumed that the material was incompressible; this assumption being justified for many soft viscoelastic materials. This assumption implies that the loss tangents in shear should be the same as in tension-compression and the magnitude of the complex modulus in shear should be one third the magnitude of the complex modulus in tension-compression. The computed values of  $G_0$  were within 20 percent of the values measured on the apparatus described herein; however, the loss tangents for the tension-compression data were 60 to 70 percent less than those measured in shear. Possibly the assumption of incompressibility was not justified. If both sets of data are correct, then Poisson's ratio must be a complex number for this material.

Calculations, not shown, were made to find the steady state temperature distribution within a typical specimen. These calculations showed the maximum temperature difference inside the specimen to be much less than  $1^{\circ}\text{C}$  for moderate shear strains at which most measurements must be taken to preserve linearity. However, for large shear strains (greater than 0.01) the temperature difference could exceed  $1^{\circ}\text{C}$  and should be considered for temperature sensitive materials such as that in Figure 6.

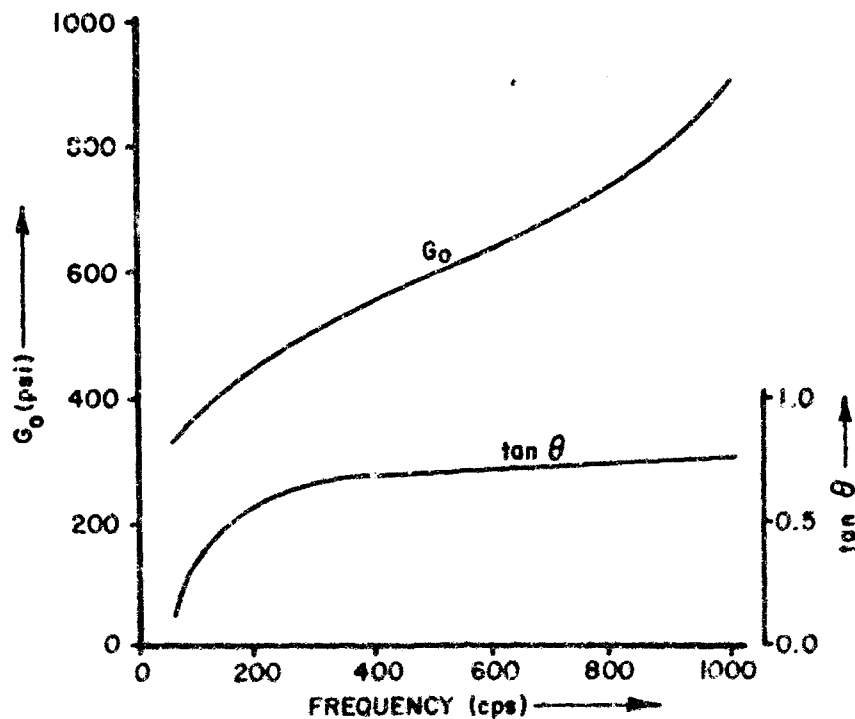


Figure 5. Dynamic Properties of Butyl Rubber as a Function of Frequency and at Room Temperature

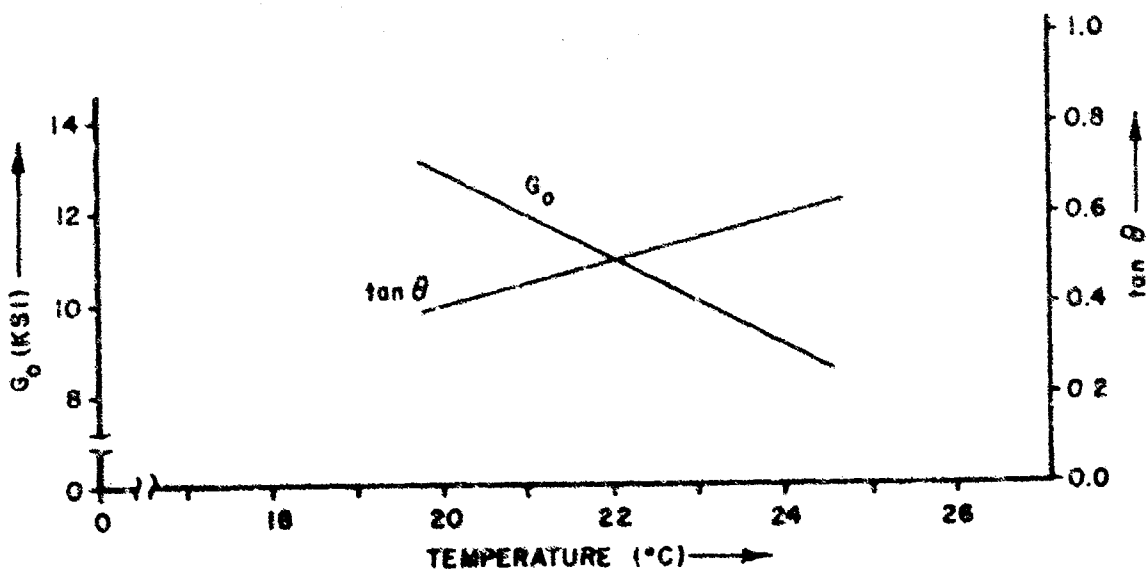


Figure 6. Dynamic Properties of an Experimental Polyurethane Damping Material as a Function of Temperature and at 1400 cps

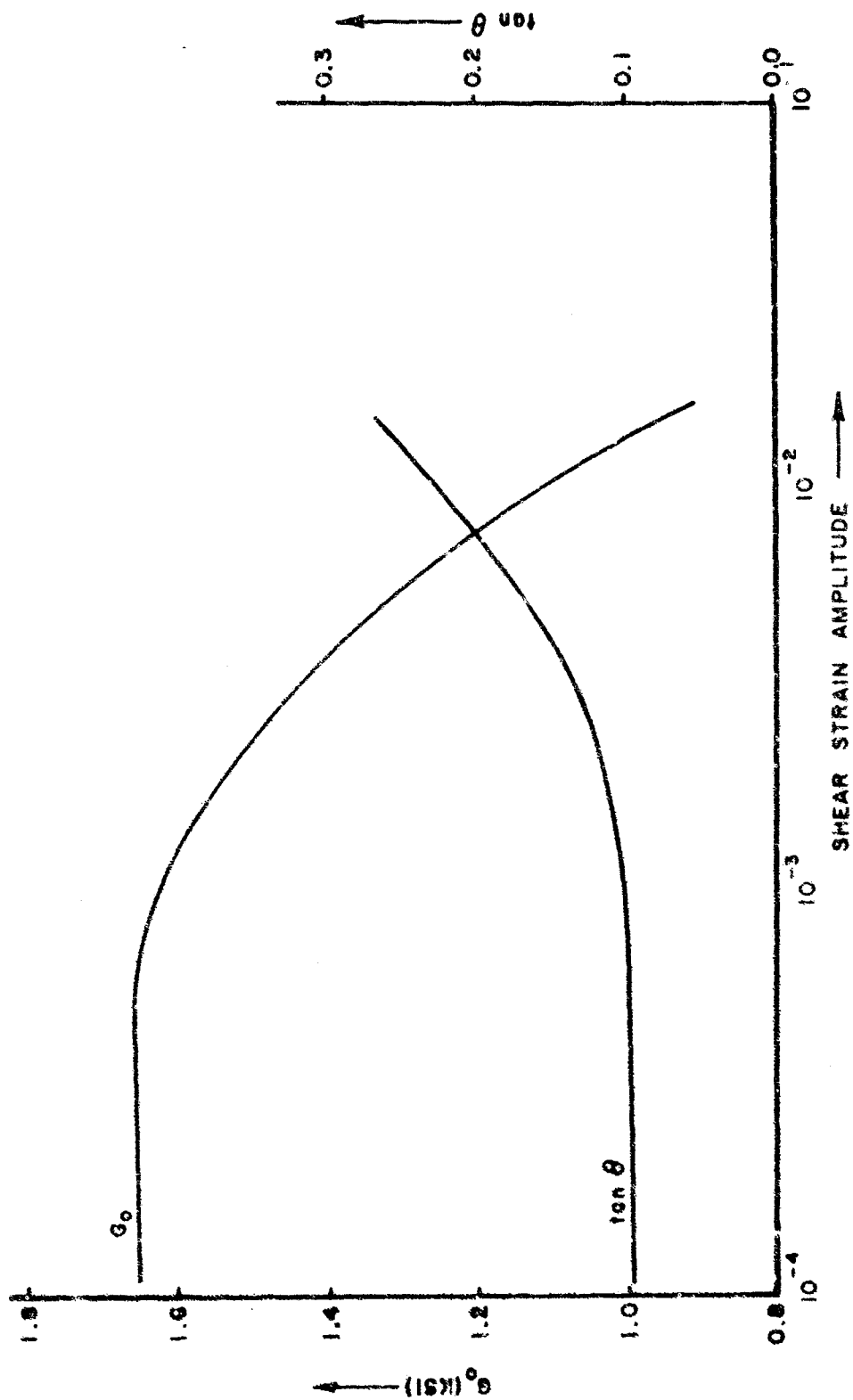


Figure 7. Dynamic Properties of a Silicone Rubber as a Function of Amplitude of Alternating Shear Strain at 1250 cps and at Room Temperature

## CONCLUSIONS

Equipment has been designed to measure the dynamic properties of viscoelastic materials in shear. A simple analysis of the response of the system has been developed. Measurements taken with this equipment show that the experimental difficulties are not extensive and the results are reliable and consistent.

For the properties of the test material to be obtained explicitly from the response of the system, the thickness of the material in shear must be such that the inertia of the specimen can be neglected. Also the experimental specimen more closely approaches the mathematical model, with a stress distribution of pure shear, as the specimen thickness decreases.

Measurements taken with this apparatus show the properties of viscoelastic materials to be sensitive to temperature, rate of deformation (that is, frequency), and amplitude of deformation. The desired environment must be produced before making the mechanical measurements.

## APPENDIX

From the experimental results obtained, a measurement taken at a relatively high frequency on a thick specimen was selected to compute the error in the approximate analysis of the response. Specifically for the butyl rubber specimen, the following were measured or computed:

Quantity	Value
$\rho$	$1.18 (10)^{-4}$ mass units**/in. <sup>3</sup>
$b$	0.13 in.
$G_0$	900 psi
$\tan \theta$	0.75
$M$	$6.34 (10)^{-3}$ mass units
$A$	6.0 in. <sup>2</sup>
$f$	1000 cps
$k_0 b$	0.294
$k^* b$	$0.279 - i 0.0930$
$\cos (k^* b)$	$0.966 + i 0.0266$
$\sin (k^* b)$	$0.277 - i 0.0895$
$\frac{\sin (k^* b)}{k^* b}$	$0.990 + i 0.00913$
$(\omega^2 M b)/(G^* A)$	$4.81 - i 3.61$

From Equations 7 and 8 the ratio of the approximate to the exact response is

$$\frac{\text{Appx.}}{\text{Exact}} = \frac{\cos (k^* b) - (\omega^2 M b)/(G^* A) \sin (k^* b)/(k^* b)}{1 - (\omega^2 M b)/(G^* A)}$$

$$= 0.995 \exp (i 0.0097)$$

This can be interpreted as -0.5 percent error in the amplitude ratio and +0.0097 radians (0.56') error in the phase angle.

\*\*No name has been assigned to the unit of mass in the inch-pound-second system of units. In this paper this unit will simply be called a mass unit.

# REFERENCES

1. J. C. Snowdon, "Representation of the Mechanical Damping Possessed by Rubberlike Materials and Structures," Journal of the Acoustical Society of America, Vol. 35, No. 6, pp 821-829, (June 1963).
2. Bernhard Gross, Mathematical Structure of the Theories of Viscoelasticity, Hermann and Company, Paris (1953).
3. W. T. Read, "Effect of Stress-Free Edges in Plane Shear of a Flat Body," Journal of Applied Mechanics, Vol. 17, pp 349-352, (December 1960).
4. T. Alfrey, Jr., "Non-Homogeneous Stresses in Viscoelastic Media," Quarterly of Applied Mathematics, Vol. 2, pp 113-119, (July 1944).
5. E. H. Lee, "Stress Analysis in Viscoelastic Bodies," Quarterly of Applied Mathematics, Vol. 13, pp 183-190, (July 1955).
6. W. T. Read, "Stress Analysis for Compressible Viscoelastic Materials," Journal of Applied Physics, Vol. 31, pp 671-674, (July 1960).
7. D. S. Berry, "Stress Propagation in Viscoelastic Bodies," Journal of the Mechanics and Physics of Solids, Vol. 6, pp 177-185, (1958).
8. R. Maxwell, "An Apparatus for Measuring the Response of Polymeric Materials to an Oscillating Strain," ASTM Bulletin No. 215, p. 76, (July 1956).

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